Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Partial Differential Equations

Subject Code: 5SC02PDE1 **Branch: M.Sc. (Mathematics)**

Semester: 2 **Date: 04/05/2017** **Time: 02:00 To 05:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Solve: $DD'z = 0$. (01)

b. Write the equation into $D\&D'$ form: $2r - 3p + 4t - 5s = 0$. (01)

c. Find a particular integral of the equation $(D^2 - D')z = e^{2x+y}$. (01)

d. Write the equation in to $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}$ form: $D(2D'^2 + D'D + 3D^2)z = 0$. (01)

e. Find the particular integral of $r - 2s + t = \sin x$. (01)

f. Find $DD'z$, if x and y in $z = z(x, y)$ is replaced by $u = \log x$ and $v = \log y$. (01)

g. $4r - s + yt - xyp + q = xy^3$ is a linear partial differential equation. Determine whether statement is True or False. (01)

Q-2 **Attempt all questions** **(14)**

a. Prove that $F(D, D')[e^{ax+by} g(x, y)] = e^{ax+by}[F(D + a, D' + b)]g(x, y)$, where a and b are constants. (05)

b. Show that if f and g are arbitrary functions of single variable, then (05)

$$u = f(x - vt + i\alpha y) + g(x - vt - i\alpha y)$$

is a solution of the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ provided that $\alpha^2 = 1 - \frac{v^2}{c^2}$.

c. Solve: $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} - 2\frac{\partial z}{\partial y} = e^{x+y}$. (04)

OR

Q-2		**Attempt all questions**	**(14)**
	a.	If $(\beta D' + \gamma)^2$ is a factor of $F(D, D')$, then prove that $e^{-\frac{\gamma}{\beta}y}[\phi_1(\beta x) + y\,\phi_2(\beta x)]$ is a solution of $F(D, D')z = 0$, where ϕ_1 and ϕ_2 are arbitrary functions of a single variable ξ.	(05)
	b.	Solve: $(D^2 + DD' - 6D'^2)z = y\cos x$.	(05)
	c.	Solve: $(D - 2D' - 1)(D - 2D'^2 - 1)z = 0$.	(04)
Q-3		**Attempt all questions**	**(14)**
	a.	Convert the equation into canonical form: $\frac{\partial^2 z}{\partial x^2} + y^2\frac{\partial^2 z}{\partial y^2} = y$.	(06)
	b.	Solve: $(x^2D^2 - y^2D'^2 + xD - yD')z = \log x$.	(06)
	c.	Classify the partial differential equation $$xyr - (x^2 - y^2)s - xyt + py - qx - 2(x^2 - y^2) = 0.$$	(02)

OR

Q-3		**Attempt all questions**	
	a.	Solve: $(x^2D^2 - 3xyDD' + 2y^2D'^2 + xD + 2yD')z = x + 2y$.	(06)
	b.	Reduce the partial differential equation $e^{2x}z_{xx} + 2\,e^{x+y}z_{xy} + e^{2y}z_{yy} = 0$ to canonical form.	(06)
	c.	Find the characteristics of $r - (2\sin x)s - (\cos^2 x)t - (\cos x)q = 0$.	(02)

SECTION – II

Q-4		**Attempt the Following questions**	**(07)**
	a.	Write one-dimensional wave equation.	(01)
	b.	What is Neumann's boundary value problem?	(01)
	c.	Write two dimensional heat equation.	(01)
	d.	Using which method one can solve second order nonlinear partial differential equation.	(01)
	e.	The solution for the Dirichlet problem for a circle of radius $a$is given by the Poisson integral formula.Determine whether the statement is True or False.	(01)
	f.	$u = x^2 - y^2$is solution of two dimension Laplace equation. Determine whether the statement is True or False.	(01)
	g.	Wave equation is considered in the Dirichlet BVP. Determine whether the statement is True or False.	(01)
Q-5		**Attempt all questions**	**(14)**
	a.	Using Monge's method, solve the equation $r - t = 0$.	(06)
	b.	Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where $u(x, 0) = 6e^{-3x}$.	(06)
	c.	What is equipotential surface?	(02)

OR

Q-5		**Attempt all questions**	**(14)**
	a.	Using Monge's method, solve the equation $5r + 6s + 3t\ + 2(rt - s^2) = -3$.	(06)
	b.	State and prove Maximum principle.	(06)

c. Write the Laplace equation in spherical co-ordinate system. (02)

Q-6 **Attempt all questions** **(14)**

a. Obtain the solution of the wave equation $\frac{\partial^2 u}{\partial r^2} + \left(\frac{1}{r}\right)\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ in the form $J_m(wr)e^{\pm i(m\theta + nz + kct)}$, where $n^2 + w^2 = k^2$. (08)

b. Show that the surfaces $(x^2 + y^2)^2 - 2a^2(x^2 - y^2) + a^4 = c$ can form a family of equipotential surfaces, and find the general form of the corresponding potential function. (06)

OR

Q-6 **Attempt all Questions** **(14)**

a. Solve the following boundary value problem in the half-plane $y > 0$, described by (08)

PDE: $u_{xx} + u_{yy} = 0, \quad -\infty < x < \infty, y > 0$

BCs : $u(x, 0) = f(x), \quad -\infty < x < \infty$,

u is bounded as $y \to \infty$, u and $\frac{\partial u}{\partial x}$ vanish as $|x| \to \infty$.

b. State and prove Harnack's theorem. (06)